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Determination of Aerodynamic Coefficients Using Accelerometer Records From a Plane Yawing Bomb

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ORDNANCE RESEARCH AND DEVELOPMENT PROJECT No. TB3-0136 AIR

BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

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B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

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October 1953

DETERMINATION OF AERODYNAMIC COEFFICIENTS USING ACCELEROMETER RECORDS FROM A PLANE YAWING BOMB

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A B E R D E E N   P R O V I N G   G R O U N D,   M A R Y L A N D

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ABSTRACT

The presently used Gavre drag functions are not well suited to the streamlined bombs dropped from high speed, high altitude airplanes. One method of obtaining better drag functions is from accelerometers mounted inside the bombs. This report develops a method of reducing such accelerometer data and applying existing exterior ballistics theories to obtain not only K_D, but K_L, K_N, K_M, K_H and spin as well. Five bombs equipped with accelerometers were dropped at White Sands Proving Ground, New Mexico during February and March 1952. Although the results were not too satisfactory they indicate the method is accurate and practical if the following can be obtained: (a) an electrical system giving a smooth record of accelerations with known limits of error, (b) accelerometers of two different ranges for small relative errors in drag, (c) accurate meteorological data near the time of drop.

LIST OF FIGURES

INTRODUCTION

In recent years higher airplane speeds and altitudes have resulted in bomb speeds greater than that of sound over an appreciable part of the trajectory. To ensure good flight at such speeds, bombs have been made more streamlined, with longer tails and fins chosen for aerodynamic qualtities. The drag coefficients of such bombs are not well represented by the coefficient corresponding to the Gavre drag function; the new bombs have less drag in the subsonic region and a steeper rise at the critical Mach number, which gives larger drags. The result is that the Bomb Ballistic Reduction Tables, based on the Gavre drag function, can only be applied to the new bombs by violent means. It is necessary to have different ballistic coefficients for range and time of flight, and to vary them with launching conditions.

The fast electronic computing machines now available make the computation of a bombing table from a given drag function rather simple. It then appears desirable to compute the table for each bomb directly, using its own drag function.

Three methods of getting the necessary aerodynamic information are available: Wind tunnel tests, spark range firings, and drops of bombs. Unfortunately, drag measurements at supersonic speeds are hard to make in a tunnel, and of lower accuracy than seems desirable. Also, wind tunnel and spark range tests must have Reynolds numbers far from those of actual flight, because small models must be used; and the effect of different Reynolds numbers ("scale effect") is not well understood. Drops of full-scale models, properly instrumented, seem to offer these advantages: (1) data are obtained from duplicates of the actual bomb, (even the surface finish, for example, is the same) dropped under tactical conditions; (2) measurements of drag, lift, moment and damping rate can be obtained from the same drop; (3) data for the construction of a bombing table should be obtainable from a small number of drops, without extensive range bombings; (4) the behaviour of the actual bomb under all expected conditions of flight can be examined directly, which gives a satisfaction that calculations from models do not provide.

It was accordingly decided to drop demolition bombs of the new family, T-54, T-55, and T-56, at White Sands Proving Ground.

A sketch of the bomb showing approximate locations and positions of the accelerometers is shown in Fig. 10.

Description of the apparatus. At the center of gravity[1] of each bomb were placed three accelerometers. The first was placed along the axis of symmetry, and indicated axial drag accelerations from zero to 2g. Its output went on Channel 1 of the telemetering apparatus. The second and third accelerometers were on the axis[1] at right angles to the first and to each other, to measure accelerations normal to the axis. Their outputs went on Channels 2 and 3. Approximately eighteen inches behind the c.g., and on the axis, were two more accelerometers on Channels 4 and 5, parallel respectively to the second and third. As far back as they could be conveniently located in the tail (h ft. from the c.g. in the text) were two more accelerometers on the axis, respectively parallel to the second and third, using Channels 6 and 7. The fourth and fifth accelerometers were carried as insurance. Estimates indicated that if undesirably large yaws developed near the speed of sound, the rearward accelerometers might be overloaded. The fourth and fifth were put in to measure such yaws, but none was observed. All accelerometers but the first had the range -1g to +1g. The first five accelerometers, the telemetering apparatus, the power supplies, and a Spheredop apparatus for position-time data were mounted on a steel tray which could be slid into the inert loaded bomb before the tail was mounted.

Ground equipment included ballistic cameras to get position and velocity at launch; Askania theodolites for position-time data during flight, in case the Spheredop didn't work well; Bowen-Knapp cameras to cover the last thousand feet of the trajectory, to give the Spheredop data a well-determined origin; telemetry receiving stations; radar tracking equipment; and weather balloons.

The telemetering and Spheredop apparatus were designed, installed, and tested by the Ballistic Measurements Laboratory of BRL; the ground equipment was operated by the Flight Determination Laboratory of WSPG; the meteorological data were furnished by the AAF weather station at WSPG. The airplane and crew were assigned by the Aberdeen Bombing Mission at Edwards AFB from the machines and crews stationed there by the AAF for bombing tests. The telemetering records were read by the Flight Determination Laboratory, and the Spheredop data were reduced by the Ballistic Measurements Laboratory.

Accelerometer records. Each accelerometer put out a voltage between 0 and 5 volts, linear in the acceleration affecting it. For drag, the output was zero for zero acceleration; for the other accelerometers the output was 2.5 volts at zero. The output controlled the pulse width in a pulse width frequency modulation telemetering circuit. Each accelerometer reading was sampled about 20 times per second. (The speed of the commutater varied somewhat.) The ground station showed the

1. The actual size of the accelerometers, and the small variations of position of c.g. from bomb to bomb, make these statements only approximately correct. No appreciable error was introduced by this approximation.

output of each accelerometer as the length of a line on an oscilloscope. The oscilloscope was photographed by a moving picture camera. Since the telemetering apparatus had thirteen channels, six were used to send reference voltages of 0, 1, 2, 3, 4, 5 volts. (A temperature indicator was used instead of 3 volts on some drops.)

The film was read by a Hermograph, which measured the length of each line by means of a photoelectric cell and marked a corresponding point on paper. The Hermograph adjusted itself automatically to the zero reference voltage line, and manual control was used to try to fit another reference voltage, 4 or 5 volts.

A section of film was read at BRL, using an ordinary reader and interpolating between the nearest two reference voltages, instead of between 0 and 4 or 5 volts. The resulting record was smoothed; but in view of the methods to be used in determining the aerodynamic coefficients, the labor of reading all the film did not seem worth while.

Conduct of the tests. Because of various delays, conflict with other programs arose, and only five bombs were dropped. The available airplane, a B-29, could not attain the desired speed and altitude. The bomb was slung below the aircraft, from which the bomb bay doors had been removed, so that most of the bomb was outside the bay. (Two 3000 lb. T-55 bombs were carried at once.) The slings were designed to release the bomb with practically no disturbance in yaw or spin. It was not possible to cock the nose up,[1] as this would have brought the tail too close to the runway. For an initial yaw, the angle of attack of the airplane and the curved airstream near the fuselage were depended on. The accelerometers normal to the axis pointed down at about 45° from the vertical. The instruments in the bomb were connected to the airplane's power supply through a pull-out plug.

Since the airfield at WSPG was small, it was necessary to fly from Edwards Air Force Base. After take-off, the airplane flew over the instrument building and the accelerometers (except the first) were read to be sure they indicated about 1g cos 45°. The airplane then flew to WSPG, reporting its arrival over Albuquerque. (From Albuquerque to WSPG the course was approximately the desired bombing course.) As the airplane approached WSPG its course was plotted on the radar plotting board and it was talked on to the desired line of flight. The bombardier picked up his target, a specially prepared circle 200 ft. in diameter. The airplane than made a large circle, giving time for the instruments in the bomb to be checked by the ground stations. About two minutes before the drop the instruments were connected to the bomb's internal power supply and their operation checked again. Last minute corrections to the line of flight were made as the airplane approached its straight bombing run, and the release of the bomb was controlled by the bombardier with a standard optical sight.[2]

---

1. As suggested by E. S. Martin, of BRL.

2. After Drop No. 3, of a 3000 lb. bomb, one engine failed, and the second bomb of the pair was dropped with only three engines working.

Remarks. The results of the experiment as shown in Section 4 were disappointing. Nevertheless, these tests did have a number of useful results. First, it was shown that the T-55 and T-56 bombs flew well, developing no objectionable yaws at speeds near that of sound. Second, the Spheredop apparatus gave good position time data. Third, methods of analyzing the data were developed which, judging from the results of these drops, will give quite accurate values of the aerodynamic coefficients if smooth data can be obtained.

1. METHOD AND THEORY

In this section, we derive the equations from which several of the aerodynamic coefficients and the spin can be determined. The coefficients corresponding to the drag D, lift L, normal force N, restoring moment M, and damping moment H will be defined by the following equations:

$$D = K_D \rho d^2 u^2, \tag{1.1}$$

$$L = K_L \rho d^2 u^2 \delta , \tag{1.2}$$

$$N = K_N \rho d^2 u^2 \delta , \tag{1.3}$$

$$M = -K_M \rho d^3 u^2 \delta , \tag{1.4}$$

$$H = -K_H \rho d^4 u \omega , \tag{1.5}$$

where ρ is the density of the air in lbs./ft.^{3}, d is the diameter of the bomb in ft., u is the air speed of the bomb in ft./sec., δ is the angle of yaw in radians, and ω the angular velocity of the longitudinal axis of the bomb in rad./sec.

Because the bombs had uncanted fins and were suspended nearly horizontally, being released with negligible angular velocity, we shall make

__Assumption 1.__ The spin rate is small.

__Assumption 2.__ The yaw is small and nearly planar.

The first of these is borne out by the data and the second is a consequence of the first. Furthermore, assumption 2 justifies making

__Assumption 3.__ The axial drag D_A represents the total drag within the accuracy of the experiment.

This assumption is borne out by calculations from the data.

Now $D_A = ma_1$ which together with (1.1) and assumption 3 give

$$K_D = ma_1/\rho d^2 u^2 \tag{1.6}$$

where m is the mass of the projectile in pounds and a_j is the acceleration as recorded on channel j, j = 1, 2, ..., 7.

In Section 3 it will be shown using the results of McShane [1] under assumptions 1, 2, 7, 8 that the differential equation of yaw δ is

$$\delta'' + 2(\alpha + u'/2u)\delta' + \beta^2\delta = 0 \tag{1.7}$$

and the yaw is given closely by

$$\delta = \sqrt{u_o/u}\, e^{-\alpha s}(ce^{i\beta s} + \tilde{c}e^{-i\beta s}) \tag{1.8}$$

where

$$\alpha = (\rho d^2/2mk^2)(K_H + k^2K_L) \tag{1.9}$$

$$\beta = \sqrt{\rho d K_M/mk^2} \tag{1.10}$$

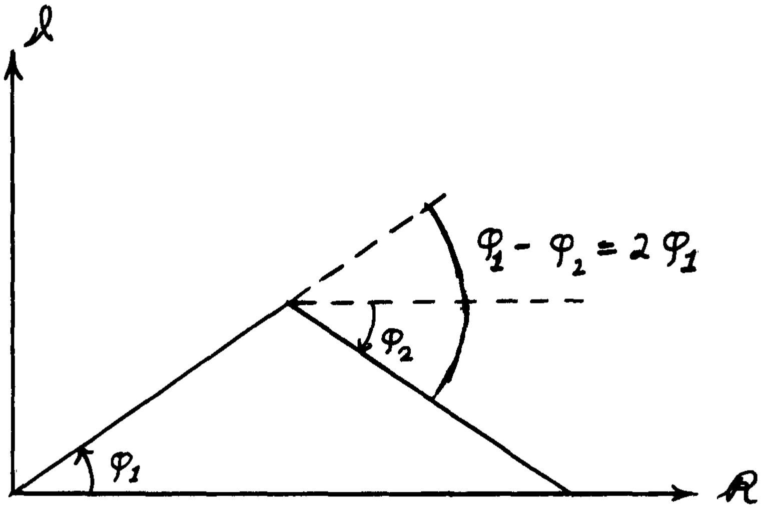
and primes indicate derivatives with respect to arclength s along the trajectory, $u = u_o$ when s = 0, c and $\tilde{c}$ are complex conjugate constants, $i = \sqrt{-1}$, B is the transverse moment of inertia, and $k = \sqrt{B/md^2}$, the radius of gyration in calibers.

Let δ_1, δ_2 denote the two complex terms of δ in (1.8) and φ_1, φ_2 the arguments of δ_1, δ_2 respectively. Then

$$\delta = \delta_1 + \delta_2\ , \tag{1.11}$$

$$\begin{cases} \varphi_1 = \beta s + \gamma\ , \\ \varphi_2 = -\beta s - \gamma\ , \end{cases} \tag{1.12}$$

where γ is the argument of c. The yaw is shown below in the complex plane.



From equations (1.12),

(1.13) $$\varphi_1 - \varphi_2 = 2\varphi_1, \qquad \varphi_2 = -\varphi_1.$$

δ is zero whenever $\varphi_1 = \pi/2 + \pi p$ for p an integer. Let $\lambda/2$ be the arc length between successive zeros of δ and call λ the wave length. Using (1.12)

$$(\beta(s + \lambda/2) + \gamma) - (\beta s + \gamma) = \pi$$

or

(1.14) $$\beta\lambda = 2\pi.$$

Substituting (1.14) into (1.10) yields

(1.15) $$K_M = \sqrt{4\pi^2 mk^2/\lambda^2 \rho d}$$

We shall call the points midway in arclength between zeros of δ the "midarcpoints" between zeros of δ. The points where $\varphi_1 = 0$, π, 2π, ... are such points. At the midarcpoints between zeros of δ, we have $\delta_2 = \delta_1$.

In terms of time derivatives (1.7) becomes

(1.16) $$\ddot{\delta} + 2\alpha\dot{\delta} + \beta^2 u^2 \delta = 0.$$

From (1.8), (1.11) we obtain

(1.17) $$\dot{\delta} = -(\alpha + u'/2u)u\delta + i\beta u(\delta_1 - \delta_2)$$

and at the midarcpoints between zeros of δ,

$$a\alpha\dot{\delta} = -2\alpha(\alpha + u'/2u)u\delta$$

which we compare with $\beta^2u^2\delta$ in (1.16). For the T55 and T56 bombs with airspeed 500 ft./sec. we obtained[1]

$$2\alpha\dot{\delta} \sim 10^{-4}\delta \ ,$$

$$\beta^2u^2\delta \sim 2.5\delta \ ,$$

$$\ddot{\delta} \sim -2.5\delta \sim -5\delta_1 \ .$$

In general we can make the

<u>Assumption 4</u>. At the midarcpoints between zeros of δ, $\dot{\delta}$ can be neglected in (1.16).

At the zeros of δ, $\delta_2 = -\delta_1$ and (1.16), (1.17) yield

$$\ddot{\delta} = -2\alpha\dot{\delta} \ ,$$

$$\dot{\delta} = 2i\beta u\delta_1 \ ,$$

or

$$\ddot{\delta} = -4i\alpha\beta u\delta_1 \ .$$

For the T55 and T56 bombs with airspeed 500 ft./sec. we obtained[1]

$$\ddot{\delta} \sim -2i \times 10^{-3}\delta_1 \ .$$

In general we can make the

<u>Assumption 5</u>. The zeros of $\ddot{\delta}$ and δ coincide within the error measurement.

Using assumptions 4 and 5 we can make the

<u>Assumption 6</u>. At the midarcpoints of the zeros of $\ddot{\delta}$, $\dot{\delta}$ can be neglected in (1.16).

Using assumption 6 we have

$$(1.18) \qquad \ddot{\delta} = -u^2\beta^2\delta$$

at the midarcpoints between zeros of δ.

---

1 For the values of the ingredients in this estimate see the discussion of Assumption 8 in Section 3.

Define the resultant r of the transverse accelerations (at the rearward accelerometers) due to yawing motion by

$$(1.19) \qquad r = \sqrt{a_6^2 + a_7^2} - \sqrt{a_2^2 + a_3^2} .$$

Because accelerations a_2 and a_3 act through the center of gravity the second term on the right in (1.19) eliminates the effect of lift upon the transverse accelerations. If accelerometers 6 and 7 are at a distance[1] h from the center of gravity,

$$(1.20) \qquad r = h\,|\ddot{\delta}| .$$

Equations (1.8), (1.18), (1.20) combine to give

$$r = h\beta^2 u_0^{1/2} u^{3/2} e^{-\alpha s} \left| c e^{i\beta s} + \tilde{c} e^{-i\beta s} \right| .$$

Let C be such a positive number that

$$(1.21) \qquad r = C u^{3/2} e^{-\alpha s}$$

for the midarcpoints between the zeros of $\ddot{\delta}$ or between the minima of r. Equation (1.21) will be used to determine α.

Now $|N| = m\sqrt{a_2^2 + a_3^2}$ which together with (1.3) yields

$$(1.22) \qquad K_N = m\sqrt{a_2^2 + a_3^2}/\rho d^2 u^2 |\delta|$$

Assumption 1 together with (1.1), (1.2) and (1.3) yield the usual formula

$$(1.23) \qquad K_L = K_N - K_D .$$

We can now determine K_H by solving (1.9)

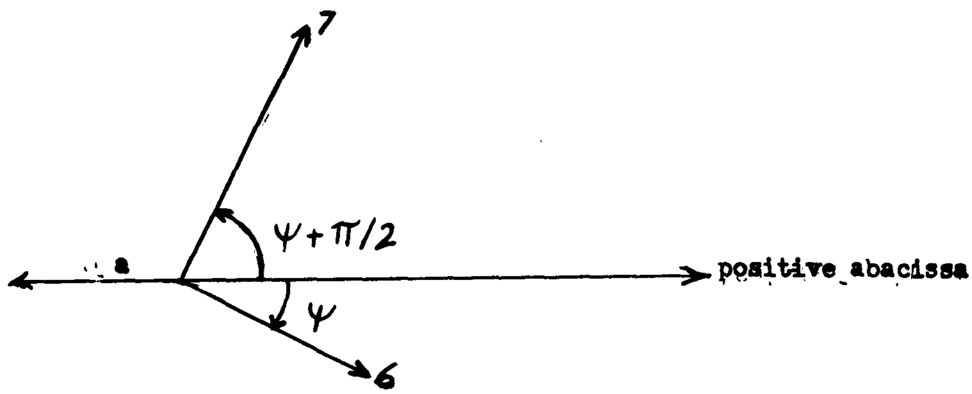
$$(1.24) \qquad K_H = (-K_L + 2m\alpha/\rho d^2)k^2$$

This analysis should be slightly modified if the bomb spins slowly, but the position and size of the maxima and minima of r will not be affected within the errors of measurement.

---

1 Accelerometers 6 and 7 may be replaced by 4 and 5 in (1.19) and in the definition of h.

To determine the axial spin choose non-rotating reference axes so that the transverse acceleration (at accelerometers 6, 7) is parallel to the axis of the abcissa. Let a be the signed magnitude of this transverse acceleration[1].



Let 6, 7 be such orthogonal axes that a_6, a_7 are the components of the above acceleration in directions 6, 7 respectively. Let ψ be the angle from the positive abcissa axis to axis 6. Then

$$(1.25) \qquad \begin{cases} a_6 = a \cos\psi \,, \\ a_7 = -a \sin\psi \end{cases}$$

and

$$(1.26) \qquad a^2 = a_6^2 + a_7^2 \,.$$

Let

$$(1.27) \qquad \begin{cases} Cs = a_6/\pm\sqrt{a_6^2 + a_7^2} \\ Sn = -a_7/\pm\sqrt{a_6^2 + a_7^2} \,. \end{cases}$$

where the sign is chosen as that of a. For the latter purpose we must keep track of the minima of a^2 (they were practically zero) which should be the same as the zeros of δ.

1 The yaw was practically plane in the part of the trajectory considered.

Combining (1.25) and (1.26) we obtain

$$(1.28) \qquad \begin{cases} \cos\psi = Cs \\ \sin\psi = Sn \end{cases}$$

or

$$(1.29) \qquad \begin{cases} \psi = \arccos Cs \\ \psi = \arcsin Sn\ . \end{cases}$$

In order to specify uniquely the terms arccos Cs and arcsin Sn of (1.29) we specify an initial ψ, require ψ be continuous, and at extrema of Cs, Sn use Sn, Cs respectively to determine whether ψ is increasing or decreasing.

The rate of spin is given by $\dot{\psi}$.

2. COMPUTATIONAL AND FITTING PROCEDURES

Required data. To determine the aerodynamic coefficients and spin of a bomb using the equations of Section 1, it is necessary to have available the physical data of the bomb (moments of inertia, mass, caliber), meteorological data (wind velocity and air density and temperature vs altitude), trajectory data (altitude and velocity vs time), and the readings of the accelerometers vs time. The air speed u can be obtained by correcting the velocity of the bomb with respect to the ground by the wind velocity. For the trajectory referred to air, the arclength, s, as a function of time, t, can be obtained by integrating the airspeed u with respect to t.

Damping rate and aerodynamic coefficients. The drag coefficient can be obtained by substitution in (1.6).

Equation (1.19) and the readings of the accelerometers can be used to determine the r vs t function. The r vs s function can then be plotted and the values of s for the minima of r (corresponding to zeros of δ and $\dot{\delta}$) can be measured. In accord with assumption 5 the measured arclengths $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_{n+1}$ from a convenient origin to the minima of r will be used to obtain fitted values $S_1, S_2, \ldots, S_{n+1}$ which differ by a uniform interval $\lambda/2$. The least square fit to the measured intervals is given by

$$(2.1) \qquad \lambda/2 = (\mathcal{S}_{n+1} - \mathcal{S}_1)/n\ .$$

Now β and K_M can be determined using (2.1) in (1.14) and (1.15), respectively. Setting

$$(2.2) \qquad S_i = S_1 + (i - 1)\lambda/2$$

for $i = 1, 2, \ldots, n+1$

we determine S_1 so that

$$\sum_{i=1}^{n+1} (\mathcal{S}_i - S_i)^2$$

is a minimum which yields

$$S_1 = -(\mathcal{S}_{n+1} - \mathcal{S}_1)/2 + \frac{1}{n+1} \sum_{i=1}^{n+1} \mathcal{S}_i \,. \tag{2.3}$$

For i = 1, 2, ..., n, let[1]

$$s_i = (S_i + S_{i+1})/2 \,. \tag{2.4}$$

From the plot of the r vs s function the values $r_1, r_2, \ldots, r_n$ at $s_1, s_2, \ldots, s_n$ (the midarcpoints between the zeros of δ) can be measured.[2]

Let α_{ij} for i,j, = 1, 2, ..., n be such a real number that

$$r_j/r_i = \sqrt{(u_j/u_i)^3} \exp\left[-\alpha_{ij}(s_j - s_i)\right] \tag{2.5}$$

where $u_1, u_2, \ldots, u_n$ are the values of u for $s_1, s_2, \ldots, s_n$. If equation (1.21) were satisfied for $s = s_i$, $r = r_i$ for i = 1, 2, ..., n, then α_{ij} would be the same for i,j = 1, 2, ..., n. Since the data contain errors, (2.5) is solved for

$$\alpha_{ij} = -\left[\log(r_j/r_i) - (3/2)\log(u_j/u_i)\right]/(s_j - s_i) \tag{2.6}$$

and the least square value α is given by

$$\alpha = \frac{2}{n(n-1)} \sum_{j=1}^{n} \sum_{i=j+1}^{n} \alpha_{ij} \,. \tag{2.7}$$

---

1 If the data are not reliable on all channels, it may be desirable to use the above procedure with one channel, say a_6, in place of r for the determination of λ/2 and $s_1, s_2, \ldots, s_n$, provided that the spin of the bomb introduces no new zeros in a_6.

2 An approximate method of making a zero-level correction on individual channels is given in the appendix.

Using (1.21) the least square value for C is given by

$$(2.8) \qquad C = \frac{1}{n} \sum_{i=1}^{n} (\mathcal{A}_i/\sqrt{u_i^3}) \exp (\alpha s_i) .$$

Fitted values $r_1, r_2, \ldots, r_n$ of the resultant acceleration due to yaw can then be obtained by substituting into (1.21),

$$(2.9) \qquad r_1 = C\sqrt{u_i^3} \exp (-\alpha s_i) .$$

For each i, $i = 1, 2, \ldots, n$, use r_i, (1.20), (1.18), (1.22) to obtain a corresponding value of $\ddot{\delta}$, δ and K_N. A least square value of K_N can then be obtained by averaging the values of K_N for $i = 1, 2, \ldots, n$.

Now use (1.23) and (1.24) to obtain K_L and K_H.

Spin. Using (1.27) we can compute the Cs vs t and Sn vs t functions. Equations (1.28) ensure that Cs and Sn are estimates of cos ψ and sin ψ. The variances of Cs and Sn become large at zeros of a. Draw smooth curves through the plots Cs vs t and Sn vs t taking into account the above property of the variances. (See Figure 5.) Henceforth we use these smoothed values of Cs and Sn. Equations (1.29) provide us with two estimates of ψ.

Let Vc, Vs be the variances of ψ as estimated by arccos Cs, arcsin Sn respectively. If these estimates were independent the best (least variance, linear combination) estimate of ψ would be

$$\psi = \frac{(\arccos Cs)/Vc + (\arcsin Sn)/Vs}{1/Vc + 1/Vs} ,$$

Further if Cs and Sn had equal variances, Vc, Vs would be approximately proportional to $1/\sin^2\psi$, $1/\cos^2\psi$, respectively. In any case a very good estimate of ψ is given by

$$(2.10) \qquad \psi = \frac{Sn^2(\arccos Cs) + Cs^2(\arcsin Sn)}{Sn^2 + Cs^2} ,$$

The rate of spin can now be obtained by differentiating numerically.

3. THE FORMULA AND DIFFERENTIAL EQUATION FOR THE YAW

The differential equation for the plane yawing of a bomb is derived by McShane as equation 4.6 of [1]. Let θ be the slope of the trajectory in a vertical plane and g be the acceleration of gravity. Then the above equation is

$$(3.1)\qquad \ddot{\delta} + (\rho d^2 u/mk^2)(K_H + k^2K_L)\dot{\delta} + (\rho dK_M u^2/mk^2)\delta$$
$$+ (\rho d^2 g/mk^2)(K_H + k^2K_D)\cos\theta - (2g^2/u^2)\sin\theta\cos\theta = 0.$$

Changing to arclength derivatives and using (1.9) and (1.10) this equation becomes

$$(3.2)\qquad \delta'' + (2\alpha + u'/u)\delta' + \beta^2\delta + (\alpha g/u^2)\cos\theta$$
$$- (2g^2/u^4)\sin\theta\cos\theta = 0 .$$

Using the notation of McShane we let $2Q_1$, Q_2, Q_3 be the coefficients of δ', δ, 1 of (3.2) and write (3.2) as

$$(3.3)\qquad \delta'' + 2Q_1\delta' + Q_2\delta + Q_3 = 0 .$$

The complete solution of (3.3) as given by (7.1) of [1] is (where σ is a bound variable replacing s and c_1, c_2 are complex constants)

$$(3.4)\qquad \delta(s) = \delta^{(r)}(s) + c_1 \exp\left[\int_0^s Q_1(\sigma)d\sigma + i\,\varphi(s)\right]$$
$$+ c_2 \exp\left[\int_0^s Q_2(\sigma)d\sigma - i\,\varphi(s)\right]$$

subject to the conditions that

$$(3.5)\qquad \varphi' = \sqrt{Q_2 - Q_1^2 - Q_1'} ,$$

$$(3.6)\qquad \varphi'' = 0 .$$

The second and third terms of (3.4) are solutions of the homogeneous equation corresponding to (3.3) and the first term is a non-oscillatory particular solution of (3.3). The latter term is called the yaw of repose and is given by (5.1) of [1].

$$(3.7) \qquad \delta^{(r)} = (2g^2/\beta^2u^4)\sin\theta\cos\theta - (\alpha g/\beta^2u^2)\cos\theta = 0.$$

For the beginning of the trajectory for the T55 and T56 bombs $\delta^{(r)}$ did not exceed 5×10^{-3} radians.[1] In general we will make the

Assumption 7. The yaw of repose can be neglected in (3.4) and (3.2).

With this assumption (3.2) becomes

$$(1.7) \qquad \delta'' + 2(\alpha + u'/2u)\delta' + \beta^2\delta = 0$$

Substituting in the values of Q_1, Q_2 into (3.5) we find

$$(3.8) \qquad \varphi' = \sqrt{\beta^2 - \alpha^2 - \alpha u'/u - (u'/2u)^2 - u''/2u}\,.$$

Now

$$u' = \dot{u}/u\ ,$$

$$u'' = \ddot{u}/u^2 - \dot{u}^2/u^3\ .$$

For the T55 and T56 bombs the following approximate values were obtained:

$u = 500$ ft./sec.	$\alpha^2 = 10^{-7}/\text{ft.}^2$
$\dot{u} = 12$ ft./sec.^{2}	$\beta^2 = 10^{-5}/\text{ft.}^2$
$\ddot{u} = 1.5$ ft./sec^3	$\alpha u'/u = 4 \times 10^{-8}/\text{ft.}^2$
$u' = 2 \times 10^{-2}$/sec.	$(u'/2u)^2 = 4 \times 10^{-10}/\text{ft.}^2$
$u'' = 6 \times 10^{-6}$/ft. sec.	$u''/2u = 6 \times 10^{-9}/\text{ft.}^2$

In general we can make the following

Assumption 8. α^2, $\alpha u'/u$, $(u'/2u)^2$, $u''/2u$ can be neglected in comparison with β^2 in (3.8).

Using this assumption (3.5) becomes

$$(3.9) \qquad \varphi' = \beta$$

and (3.6) is satisfied. Integrating (3.9) we obtain

$$(3.10) \qquad \varphi(s) = \beta s + \bar{\gamma}$$

[1] For the values of the ingredients in this estimate see the discussion of Assumption 8 in Section 3.

where γ is a real constant. Further

$$(3.11) \qquad \exp \int_0^s Q_1(\sigma)d\sigma = \sqrt{u_o/u}\, e^{\alpha s} .$$

We can take our reference frame so that the yaw δ is real. Then c_1, c_2 of (3.4) are complex conjugates. Using (3.10), (3.11) and assumption 7 equation (3.4) becomes

$$(1.8) \qquad \delta = \sqrt{u_o/u}\, e^{-\alpha s}(ce^{i\beta s} + \tilde{c}e^{-i\beta s})$$

where c and $\tilde{c}$ are complex conjugates and $c = c_1 \exp i\gamma$, $\tilde{c} = c_2 \exp(-i\gamma)$.

4. RESULTS AND CONCLUSIONS

As remarked in the Introduction, the experiment was not altogether successful, for the aerodynamic coefficients obtained by the methods discussed in Sections 1 and 2 are for the most part of doubtful accuracy.

Two reasons for the poor results can be attributed directly to the electrical equipment used. First, it was necessary to measure drag from accelerometers of greater range (and consequently of less accuracy at lower speeds) than had been planned. Second, the transverse accelerometers showed rapid and spectacular fluctuations (cf. Figures 1 and 2), which made the analysis difficult and introduced large relative errors into the computations.

Because the yaws were small, and damped out fairly quickly, the aerodynamic coefficients (except K_D) and the spin were composed only for the first part of the trajectory, at a Mach number of about 0.6.

The following discussion of the data for each drop is given to illustrate the difficulties in analysis mentioned in the preceding paragraph, how some of the difficulties were overcome, and their effect upon subsequent computations.

Drop 1. The accelerometer corresponding to Channel 1 did not function at all, so the most important data were missing. No attempt was made to reduce the data from other channels.

Drop 3. The data must be considered as no better than fair. Although Channel 6 displayed fairly well defined extrema and zeros, Channel 7 (see Figure 2) fluctuated up and down widly, rendering the data useless. Instead, Channels 4 and 5 were used. These channels gave much better curves, but the small amplitudes decreased the relative accuracy of the readings. Channels 2 and 3 presented fairly good curves.

Drop 4. Because Channel 7 did not function, the data are useless for computations other than K_D, K_M and λ. Channel 6 was fair; from it alone λ was determined and K_M was computed. An attempt to use Channels 4 and 5 in place of Channels 6 and 7 produced meaningless results.

Drop 2. On the whole, the data are very good. Channel 7 has some wild fluctuations downward, but if these fluctuations are considered as error and ignored, the curve has well defined extrema and zeros. The other channels are very good.

Drop 5. The data are only fairly good. Channel 6 is good, but Channel 7 has very wild downward fluctuations - more than Channel 7 of Drop 2. The upper envelope of Channel 7 seemed to be in phase with Channel 6 and this envelope was used to determine r. Use of this envelope, however, makes it impossible to estimate the error in Channel 7. Thus there are unknown errors in K_N, K_L, and K_H introduced through unknown errors in r and α. Channel 3 is very good. Channel 2 has wild downward fluctuations, but again the upper envelope was used.

Figures 3 and 4 show the computed values of K_D vs Mach number for the T55 and T56 bombs respectively. The solid curve of each figure is one probable K_D curve defined by the data, faired in by eye. The broken curves of Figures 3 and 4 represent the error in K_D due to an error of 0.02 units of gravity which is the probable maximum error in the smoothed a_1 vs t curves. It was assumed that the zero level, after correction, is within 0.01g of the true zero and that the curve drawn represents the acceleration (uncorrected for zero level) within 0.01g. An appreciable error in K_D could result from an error in ρ. In some cases the meteorological data were not taken until several hours after the drop. Thus, no satisfactory estimate of the error in ρ can be made. If the determination of u were in error by 10 ft./sec., at 500 ft./sec. this would introduce into K_D an error of only 4%, a small fraction of the error introduced by an error in a_1. As us increases, this error becomes rapidly less important, and hence K_D is relatively unaffected by errors in u.

Using the curves of K_D in Figures 3 and 4, Mr. E. S. Martin, of these laboratories, has kindly computed the trajectories of a number of bombs which were dropped in the range bombing work at Edwards AFB. Thus the range and time of flight could be compared with the observed values. It appears that the values near a Mach number of 1.1 are probably too low for both the 3,000 lb. T55 and 10,000 lb. T56 bombs.

The computed values of K_H, K_M, K_L and K_N for each drop are given in Table 1. Although it was impossible to estimate the errors in these coefficients, a few general remarks can be made. The accuracy of K_M is poor because a_2 and a_3 are small. Even though in general these coefficients as given in Table 1 are no better than an indication of the order of magnitude, those for Drop 2 are probably of useful accuracy.

The angle of rotation ψ and the rate of spin $\dot{\psi}$ for Drops 2 and 3 are shown in Figures 6 through 9, the optical and Doppler values of ψ (adjusted for phase differences) being shown for comparison. The bomb of Drop 5 displayed no discernible rotation. The spin rate 8 seconds after release is given in Table 1.

Table 1

Drop No.	3	4	2	5
Bomb Type	T55	T55	T56	T56
Serial No.	6164	6169	6116	6132
Date Dropped	4 Mar 52	4 Mar 52	25 Feb 52	5 Mar 52
m (lbs.)	2965.5	2989.5	10932	10900
d (ft.)	2	2	3.833	3.833
B (lb.-ft.^{2})	23083	22229	99097	101947
h (ft.)	1.55		5.57	5.57
α	.000296	——[1]	.000229	.000338
β	.00499	——[1]	.00416	.00397
$\lambda/2$ (ft.)	630	744	756	791
K_D (Mach .6)	.0845	.0845	.0495	.0495
K_M	2.70	1.72	1.23	1.14
K_N	0.71	——[1]	1.51	3.20
K_L	0.63	——[1]	1.46	3.15
K_H	30.9	——[1]	7.6	10.7
$\dot{\psi}$ (after 8 sec.) (rad./sec.)	1.11	——[1]	.175	0

[1] Channel 7 did not function.

Comparison with Cornell data. For purposes of comparison, K_M and K_N were computed from Cornell Aeronautical Laboratory [2]. The Cornell report gave graphs of C_M and C_N vs α (here α is the angle of attack in degrees) for the standard 3,000 and 10,000 lb. bombs. Now C and K are related by

$$(4.1) \qquad \begin{cases} K_M = -(180/\delta)(dC_M/d\alpha)_0 \\ K_N = (180/\delta)(dC_N/d\alpha)_0 \end{cases}$$

where the subscript zero indicates that the derivative is evaluated for zero degrees angle of attack. If C_M and C_N are defined as least-squares cubic equations in α, the values of K_M and K_N can be computed directly from (4.1). The positions of the centers of gravity of the bombs dropped at White Sands differed by about 0.1 calibers from those of the standard bombs for which the Cornell data were computed. Hence, equations (4.1) must be multiplied by the ratio of the c.g. distances. Table 2 is a comparison of our results with the adjusted Cornell results for Mach number 0.6. The superscripts o and c indicate our results and the adjusted Cornell results respectively.

Table 2

Drop No.	3	4	2	5
K_M^o	2.70	1.72	1.23	1.14
K_M^c	2.40	2.42	1.25	1.24
K_N^o	0.71	--	1.51	3.20
K_N^c	1.17	1.17	1.52	1.52

From the agreement between the results of Drop 2 and those of the Cornell data it can not be concluded that the scale effect is negligible for this bomb, even though the accelerometer data are quite smooth, because the sample of good data is too small.

Conclusions. Our results indicate that this method of getting aerodynamic data is accurate and practical if the following improvements can be made:

(a) The electrical system can be made to give a smooth record of the accelerations, with fairly well-known limits of error. This is the most important.

(b) The drag measurements can be made with two or more accelerometers of different ranges, to obtain small relative errors with low drag.

(c) Accurate meteorological data can be given near the time of the drop.

If K_M, K_H, etc., are wanted for Mach numbers much higher than that of release, it may be necessary to excite oscillations late in the flight by some device.

J. CONLAN

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L. G. MAYNARD

APPENDIX: APPROXIMATE ZERO-LEVEL CORRECTION

If the airplane is not accelerating, the portion of the longitudinal acceleration curve before the time of release should be zero. With this assumption, the smoothed a_1 vs t curve to the left of the time of release indicates the true zero level of a_1, and any needed zero level correction can be made.

If the indicated zero for a particular transverse acceleration j is in error, a corrected zero can be found approximately assuming the component a_j behaves like r. Let $a_j(s_i)$ and $\mathcal{a}_j(s_i)$ be the corrected and indicated accelerations, respectively, at s_i. We attempt to determine k_j so that

$$\text{(A1)} \qquad a_j(s_i) = \mathcal{a}_j(s_i) - k_j \, .$$

Using (1.18) and the equal spacing of $s_1, s_2, \ldots, s_n$ we have

$$\text{(A2)} \qquad \left[u_i/u_{i+1}\right]^{-3/2} a_j(s_i)/a_j(s_{i+1})$$
$$= \left[u_{i+1}/u_{i+2}\right]^{-3/2} a_j(s_{i+1})/a_j(s_{i+2})$$

for $i = 1, 2, \ldots, n-2$. Making the approximation

$$u_{i+1}{}^2 = u_i u_{i+2}$$

in (A2) we obtain

$$\text{(A3)} \qquad a_j(s_i)/a_j(s_{i+1}) = a_j(s_{i+1})/a_j(s_{i+2})$$

or using (A1) in (A3),

$$\text{(A4)} \qquad \left[\mathcal{a}_j(s_i) - k_j\right]\left[\mathcal{a}_j(s_{i+2}) - k_j\right] = \left[\mathcal{a}_j(s_{i+1}) - k_j\right]^2$$

for $i = 1, 2, \ldots, n-2$. The least square solution of (A4) for k_j is

$$\text{(A5)} \qquad k_j = \sum_{i=1}^{n-2} p_i q_i \Big/ \sum_{i=1}^{n-2} p_i{}^2$$

where

$$p_i = a_j{}^2(s_{i+1}) - a_j(s_i)a_j(s_{i+2})$$

and

$$q_i = 2a_j(s_{i+1}) - a_j(s_i) - a_j(s_{i+2}).$$

List of Symbols

a_j = acceleration as recorded on channel j, $j = 1, 2, \ldots, 7$.

a = transverse acceleration, $a^2 = a_6^2 + a_7^2$.

B = transverse moment of inertia of bomb.

Cs = $a_6/\pm\sqrt{a_6^2 + a_7^2}$, sign corresponding to that of a.

d = diameter of bomb.

g = acceleration due to gravity.

h = distance from center of gravity of bomb to accelerometers.

k = radius of gyration of bomb $= \sqrt{B/md^2}$.

m = mass of bomb.

n = the number of intervals defined by successive zeros of δ.

r = resultant acceleration due to yaw $= \sqrt{a_6^2 + a_7^2} - \sqrt{a_2^2 + a_3^2}$.

s = distance along the trajectory.

/ Sn = $-a_7/\pm\sqrt{a_6^2 + a_7^2}$, sign corresponding to that of a.

t = time.

u = air speed of bomb.

α = $(\rho d^2/2mk^2)(k^2 K_L + K_H)$.

β = $\sqrt{\rho d^3 K_M/B}$.

δ = yaw.

δ_1, δ_2 = two complex terms of δ in equation (1.8).

θ = slope of the trajectory in a vertical plane.

$\lambda/2$ = distance between successive zeros of δ.

ρ = air density.

φ = βs + (real constant).

φ_1 = argument of δ_1, precession.

φ_2 = argument of δ_2, mutation.

ψ = angle of axial rotation of the bomb.

ω = angular velocity of the longitudinal axis of the bomb.

A dot ($\dot{}$) denotes a derivative with respect to time, and a prime ($'$) denotes a derivative with respect to arc length.

AERODYNAMIC COEFFICIENTS

K_D = drag coefficient.

K_H = damping moment coefficient.

K_L = lift coefficient.

K_M = restoring moment coefficient.

K_N = normal force coefficient.

AERODYNAMIC FORCES

D = drag force = $K_D \rho d^2 u^2$.

H = damping moment = $-K_H \rho d^4 u\omega$.

L = lift force = $K_L \rho d^2 u^2 \delta$.

M = restoring moment = $-K_M \rho d^3 u^2 \delta$.

N = normal force = $K_N \rho d^2 u^2$.

The following special symbolism regarding arc length and acceleration is used in Section 2.

1. Capital letters refer to values of the functions at the zeros of δ.

2. Small letters refer to values of the functions at the extrema of δ.

3. Script letters refer to observed or unfitted values.

4. Printed letters refer to fitted values.

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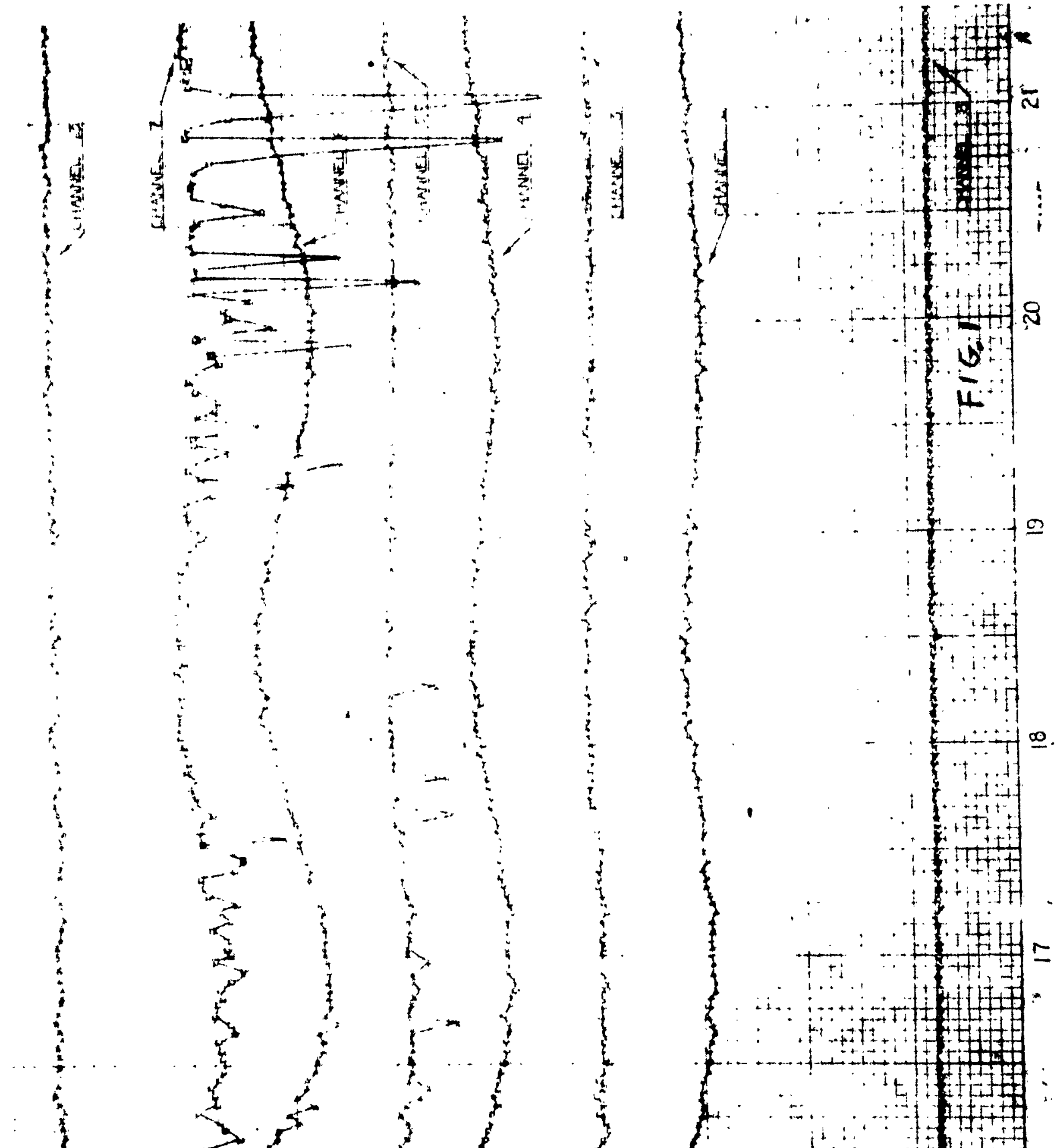


FIG. 1

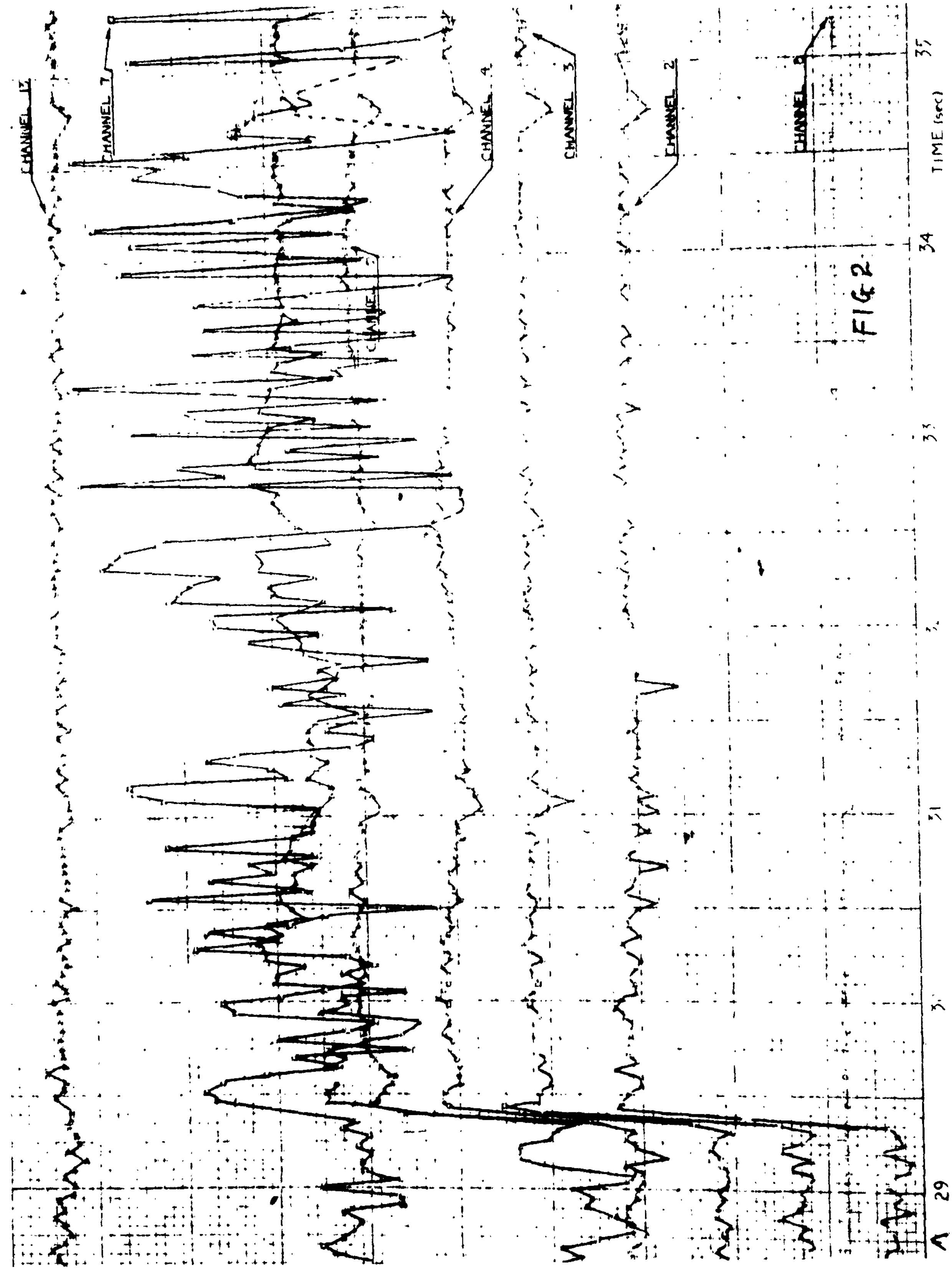
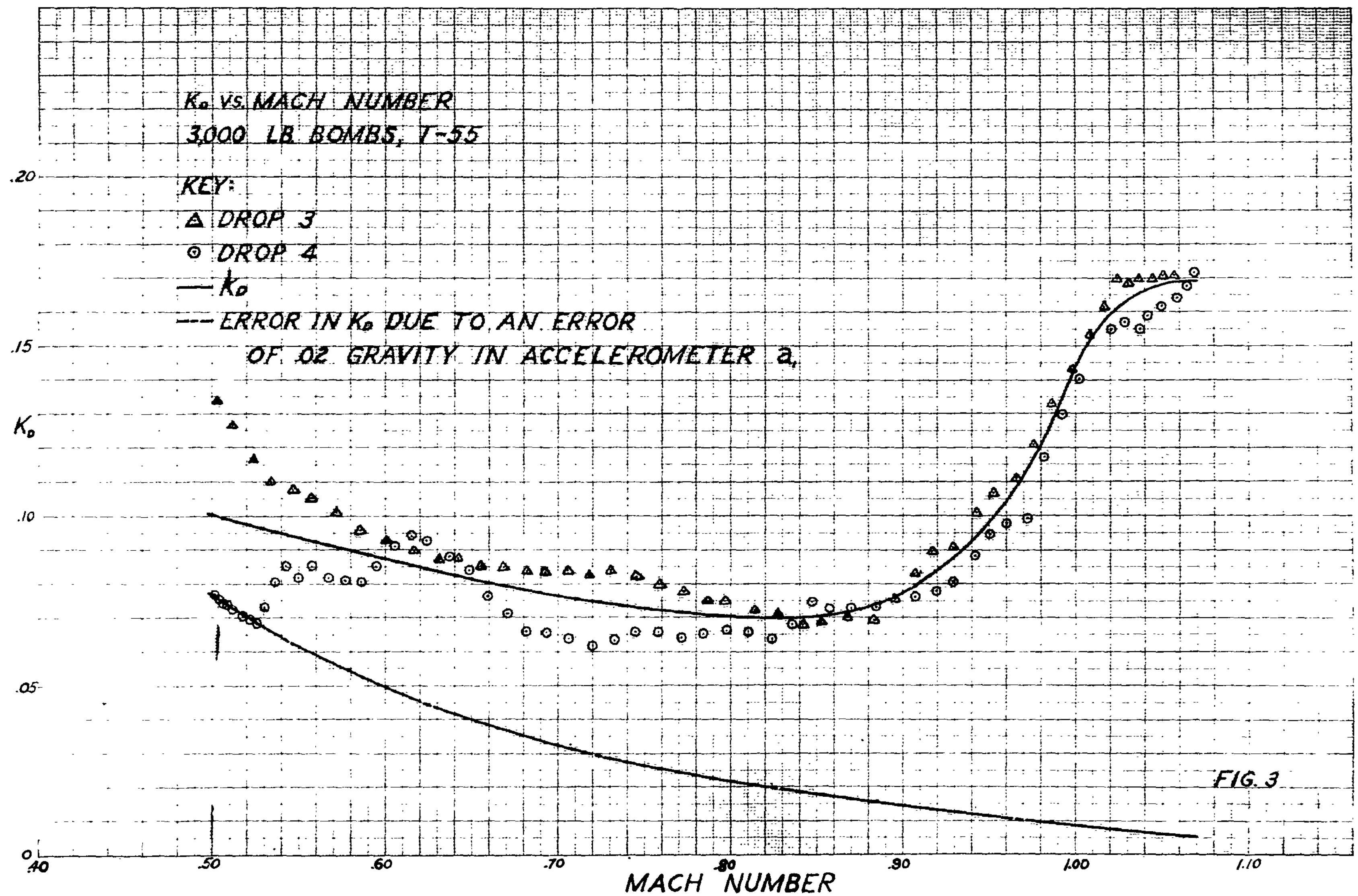
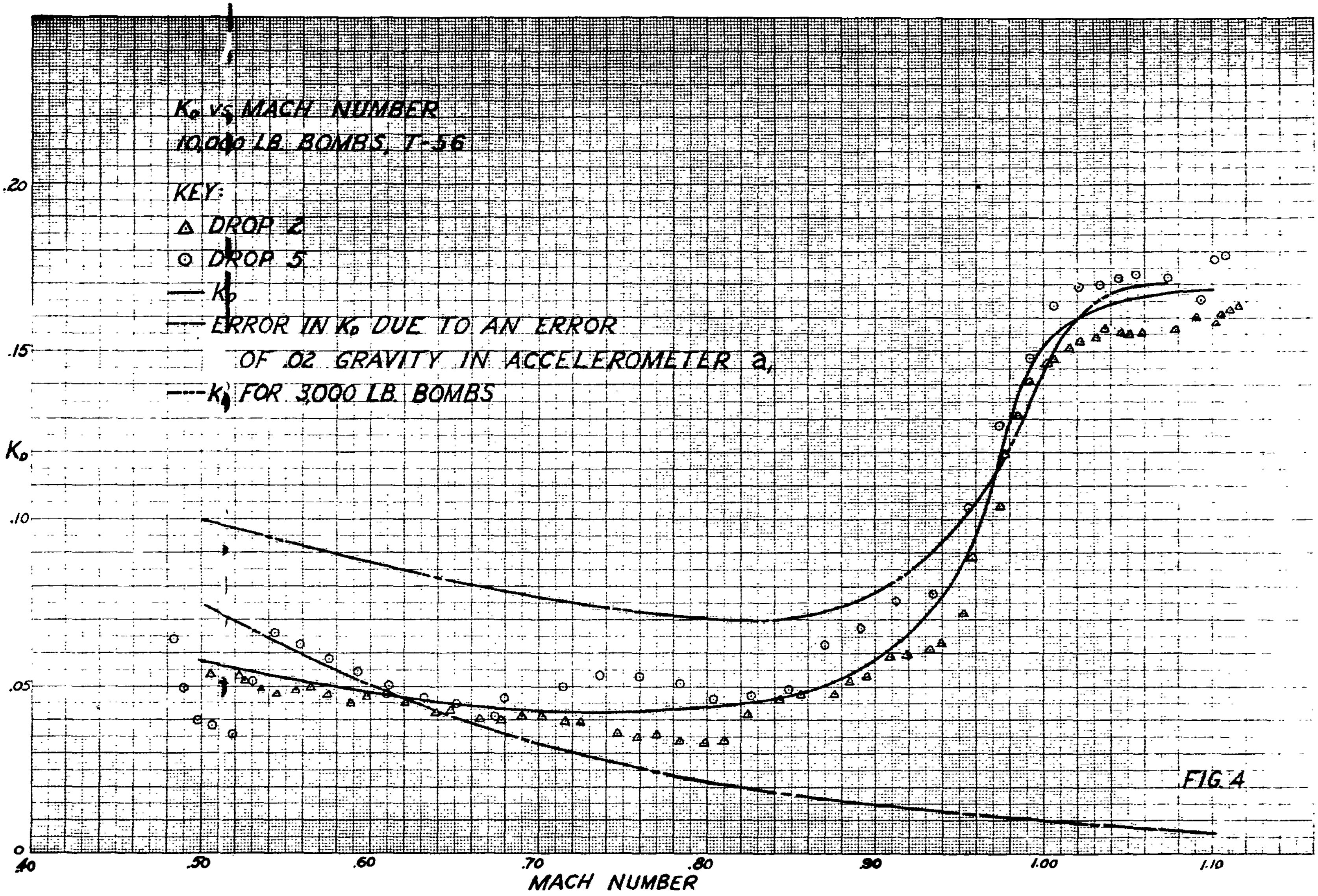


FIG 2



K_D vs. MACH NUMBER
3,000 LB BOMBS, T-55
KEY:
DROP 3
DROP 4
K_D
ERROR IN K_D DUE TO AN ERROR OF .02 GRAVITY IN ACCELEROMETER a_t
.20
.15
K_D
.10
.05
0
.40
.50
.60
.70
.80
.90
1.00
1.10
MACH NUMBER

FIG. 3



K_D vs MACH NUMBER
10,000 LB. BOMBS, T-56
KEY:
△ DROP 2
⊙ DROP 5
— K_D
— ERROR IN K_D DUE TO AN ERROR OF .02 GRAVITY IN ACCELEROMETER a_1
---- K_D FOR 3,000 LB. BOMBS
K_D
.20
.15
.10
.05
0
.40
.50
.60
.70
.80
.90
1.00
1.10
MACH NUMBER

FIG. 4

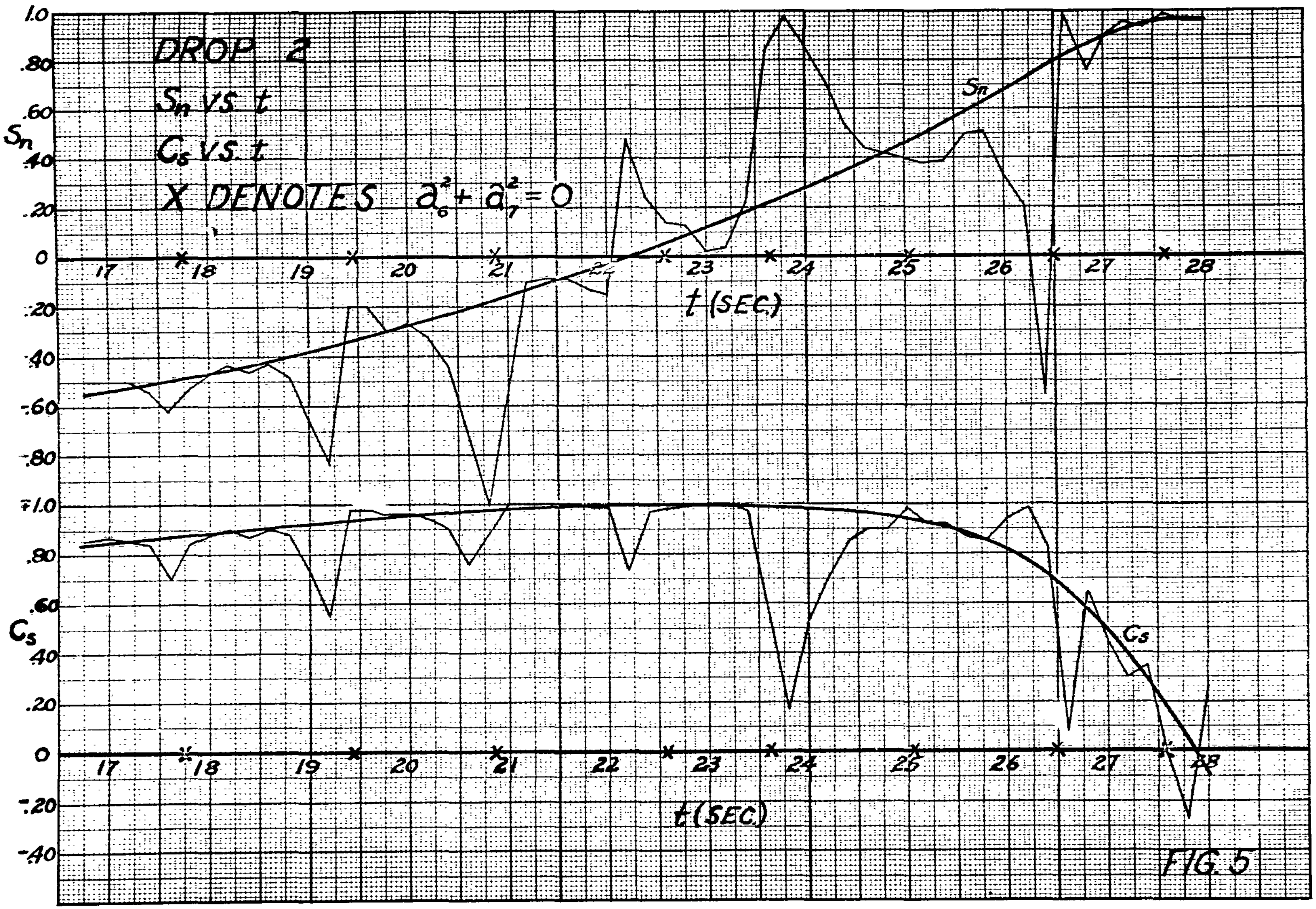
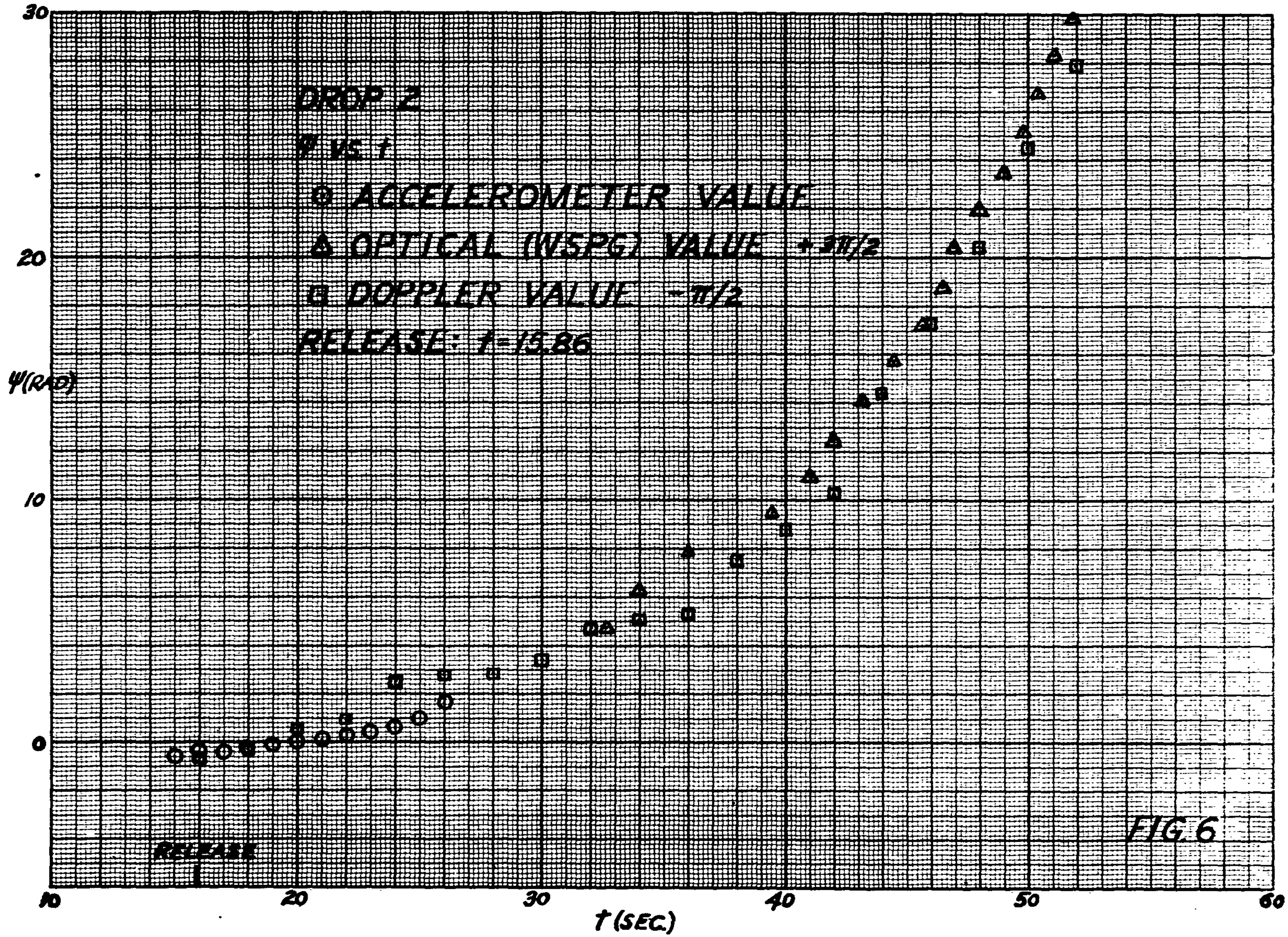

DROP 2
S_n VS t
C_s VS t
X DENOTES $a_6^2 + a_7^2 = 0$
S_n
C_s
t (SEC)
1.0
.80
.60
.40
.20
0
-.20
-.40
-.60
-.80
∓1.0
17
18
19
20
21
22
23
24
25
26
27
28

FIG. 5



DROP 2
Ψ VS. t
⊙ ACCELEROMETER VALUE
△ OPTICAL (WSPG) VALUE +3π/2
⊡ DOPPLER VALUE −π/2
RELEASE: t=15.86
Ψ(RAD)
30
20
10
0
RELEASE
10
20
30
40
50
60
t (SEC.)

FIG. 6

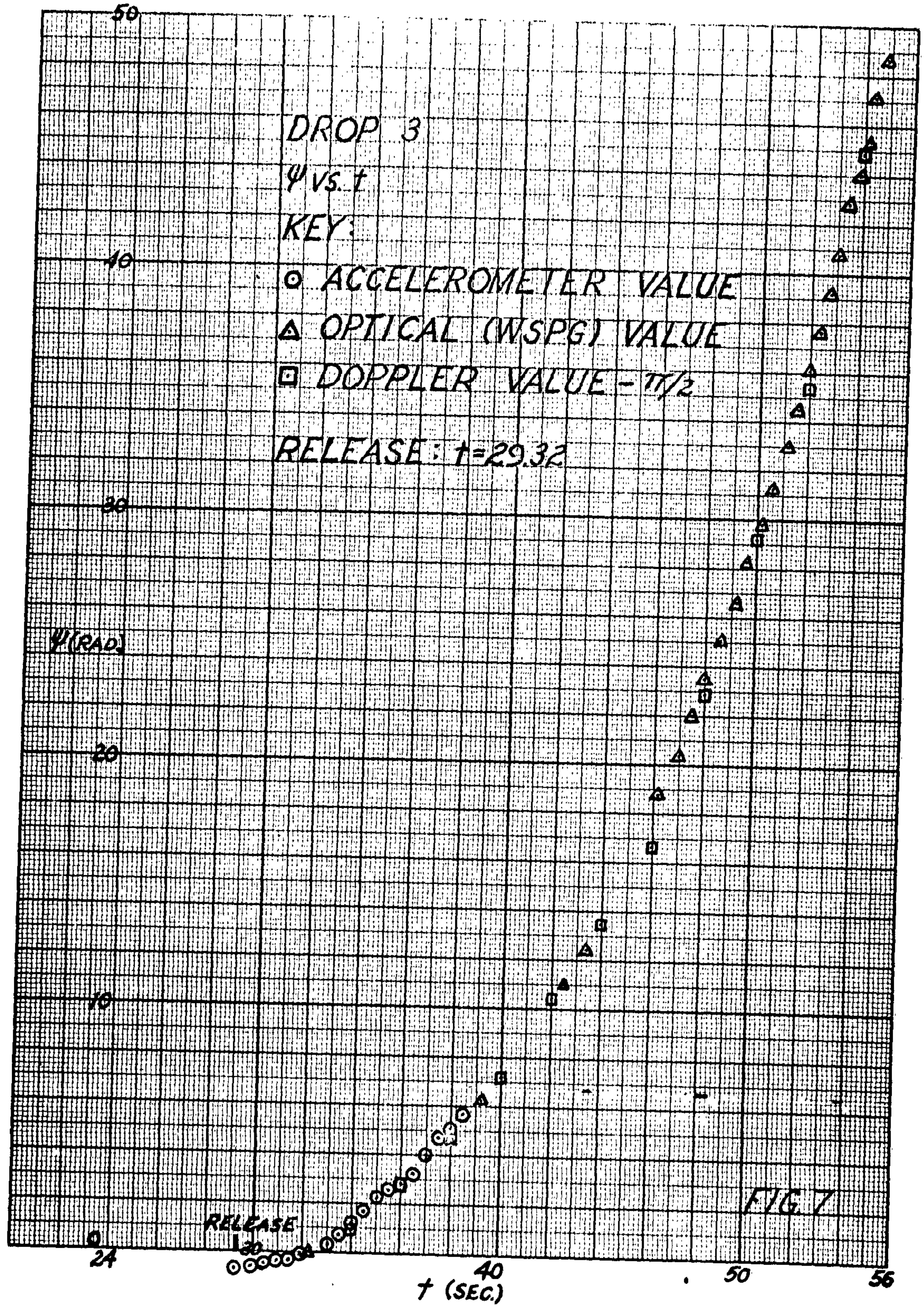
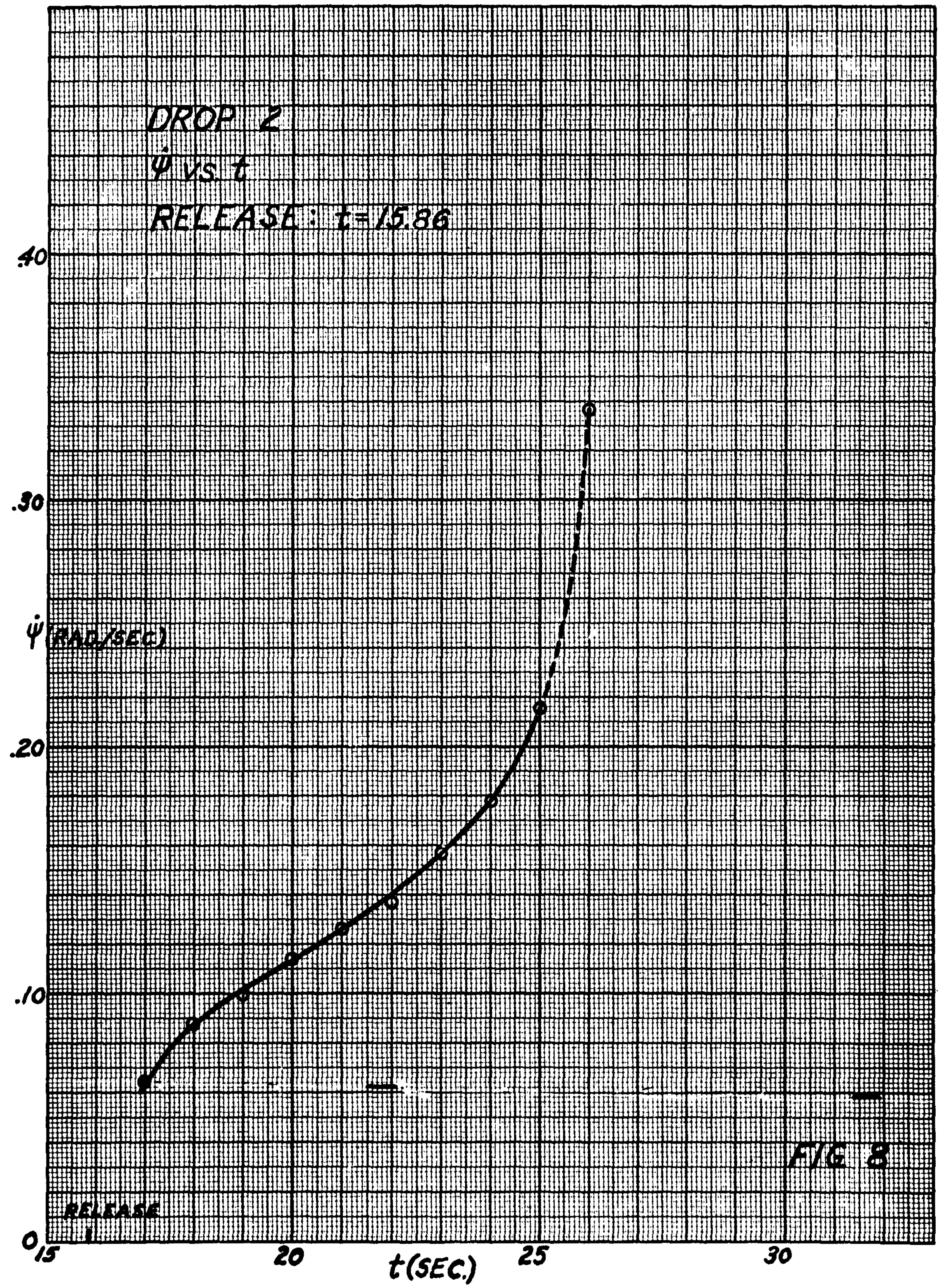

DROP 3
ψ VS. t
KEY:
ACCELEROMETER VALUE
OPTICAL (WSPG) VALUE
DOPPLER VALUE − π/2
RELEASE: t=29.32
ψ (RAD)
t (SEC.)
RELEASE
50
40
30
20
10
0
24
30
40
50
56

FIG. 7


DROP 2
$\dot{\psi}$ VS. t
RELEASE: t=15.86
.40
.30
$\dot{\psi}$ (RAD./SEC.)
.20
.10
0
RELEASE
15
20
25
30
t (SEC.)
FIG 8

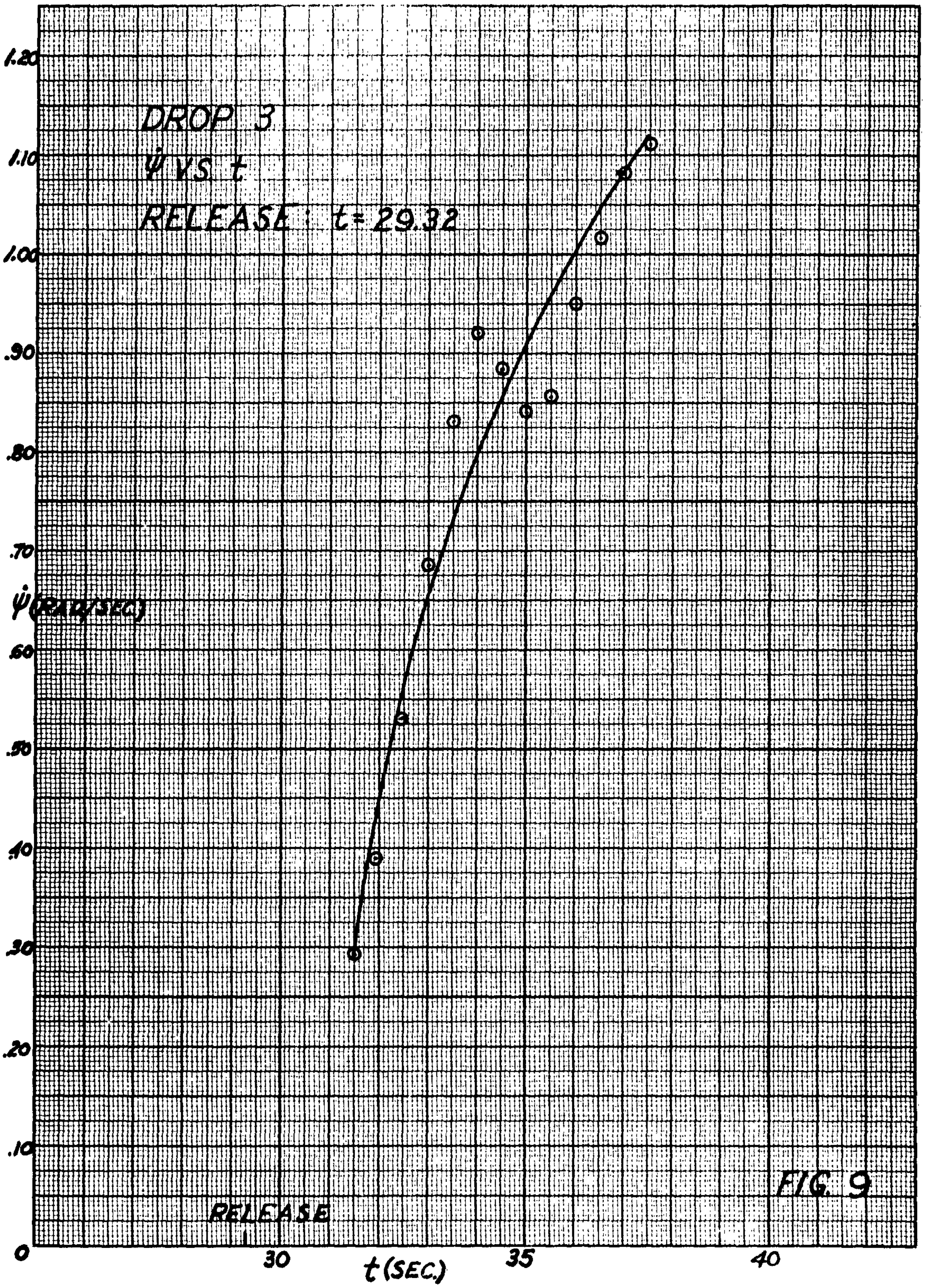
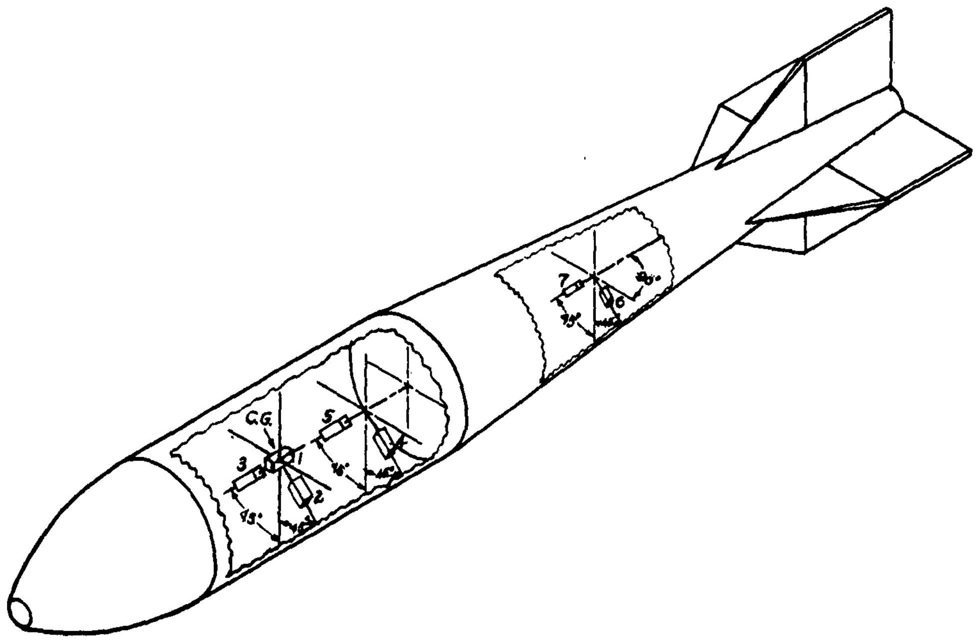

DROP 3
ψ̇ VS. t
RELEASE: t = 29.32
ψ̇ (RAD/SEC)
1.20
1.10
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
0
RELEASE
30
35
40
t (SEC.)

FIG. 9



SKETCH OF BOMB SHOWING APPROXIMATE
LOCATIONS AND POSITIONS OF ACCELEROMETERS

FIG. 10

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